# **Challenges in Comparing Numerical Solutions** for Optimum Weights of Stiffened Shells

Satchi Venkataraman\*

San Diego State University, San Diego, California 92182

Luciano Lamberti<sup>†</sup>

Politecnico di Bari, Bari 10126, Italy

Raphael T. Haftka<sup>‡</sup>

University of Florida, Gainesville, Florida 32611-6250

and

Theodore F. Johnson<sup>§</sup>

NASA Langley Research Center, Hampton, Virginia 23681-2199

Optimizations of stiffened shells with different stiffener shapes performed to rank and identify the optimum designs during the preliminary design trade studies require a large number of analyses and hence rely on the use of efficient but approximate analysis methods. In the design of shells, the treatment of imperfections on buckling loads and stresses is of paramount importance. It is demonstrated how conservativeness of the approximate analyses used in buckling load calculation, the number of variables optimized (design freedom), and nonstructural constraints influence the weight of optimum designs. This demonstration is based on the results of a trade study performed to compare minimum weight designs of stiffened shells optimized under stress and buckling constraints for a reusable launch vehicle tank. PANDA2 was selected for the present study because it uses approximate analysis procedures that permit the many thousands of structural analyses needed for global optimization and it also has sophisticated machinery for generating imperfections and accounting for their effects. Optimum weights were influenced not only by material choice, number of optimization variables, and manufacturing constraints, but also by the analysis model conservativeness. Optimization of shells with effect of initial imperfections exhibited substantial weight differences between different stiffened-shell concepts, partly because of conservativeness in the analysis.

#### Nomenclature

=	thickness of sandwich honeycomb core, in.
=	diameter of the hexagonal honeycomb core
	cell, in.
=	elastic modulus of the honeycomb core normal
	to its plane, psi
=	elastic modulus of the lamina in the fiber
	direction, Msi
=	elastic modulus of the lamina in the transverse
	to fiber direction, Msi
=	core-crushing strength normal to its plane, psi
=	shear strength of honeycomb core along the
	ribbon direction, psi
=	shear strength of the honeycomb core transverse
	to ribbon direction, psi
=	ultimate tensile strength of the honeycomb core
	material, psi
=	shear modulus of the sandwich honeycomb core
	in the $x$ – $z$ plane, psi
=	shear modulus of the sandwich honeycomb core
	in the <i>y</i> – <i>z</i> plane, psi
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<sup>\*</sup>Assistant Professor, Department of Aerospace Engineering. Member AIAA.

$m_{i,i=1,2,,3}$	=	subscript indices used to denote optimum
		number of plies at symmetry position in the
		laminate description
$N_x$	=	axial force resultant acting on the shell wall,
		lbf/in.
$n_c$	=	number of stiffened-panel concepts optimized
$n_d$	=	average number of design variables optimized
		for each stiffened-panel concept
$n_g$	=	maximum number of line searches performed
•		in PANDA2 global optimization
$n_{i,i=1,2,,5}$	=	subscript indices used to denote optimum
, , , , , , , , , , , , , , , , , , , ,		number of plies in laminate description
		for a given direction
$n_s$	=	number of PANDA2 stiffened-panel analysis
		per search
$n_{v}$	=	number of PANDA2 analysis models required
U		of each stiffened-panel concept
S	=	honeycomb core cell size
$t_c$	=	wall thickness of the hexagonal honeycomb core
-		cell, in.
$ ho_c$	=	density of the honeycomb core material, lb/in. <sup>3</sup>
$\rho_c'$	=	apparent density of the honeycomb core, lb/in. <sup>3</sup>
r c		T. F

in-plane shear modulus of the lamina, Msiout-of-plane shear modulus of the lamina, Msi

# Introduction

ESIGN trade studies that compare stiffener geometry and wall constructions of stiffened shells for different design applications have been previously published (e.g., Agarwal and Sobel, Swanson et al., and Budiansky). Such studies are often performed in the preliminary stage of designing new aerospace vehicles to understand the implications of geometry and/or loads, mission requirements, and material choices on the structural weight. Trade studies include a large number of design optimizations to investigate the effect of parameters, such as stiffener geometry (Fig. 1), material selection, and the type of construction and manufacturing

<sup>&</sup>lt;sup>†</sup>Faculty, Dipartimento di Progettazione e Ingegneria Meccanica e Gestionale

<sup>&</sup>lt;sup>‡</sup>Distinguished Professor, Department of Mechanical and Aerospace Engineering; haftka@ufl.edu. Fellow AIAA.

<sup>§</sup> Aerospace Technologist, Mechanics and Durability Branch. Member AIAA.

Table 1 Estimated number of analyses for stiffened-panel trade study

Analyses	No.
Number of stiffened-panel concepts $(n_c)$	14
Variations of each concept $(n_v)$	2
Average number of design variables $(n_d)$	10
Number of line searches performed in	275
PANDA2 global optimization $(n_g)$	
Number of analyses per search $(n_s)$	11
Total number of analyses performed	84,700
$(n_c \ n_v \ n_g \ n_s)$	

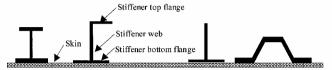


Fig. 1 Schematic of T, J, blade, and hat stiffeners.

methods selected, on the optimum structural weight. Even with a small number of choices for each component just mentioned, a large number of possibilities exist. A systematic search of the design space using global optimization of each design concept requires numerous analyses. Table 1 provides an estimate of the number of analyses required for the reusable launch vehicle (RLV) trade study reported here. Constraints on computational resources and design cycle time necessitate the use of efficient and inexpensive analysis methods for design optimization.

The shell design code PANDA2, developed by Bushnell<sup>4</sup> at Lockheed Martin Co., was chosen for this work as it provided computationally inexpensive estimates of buckling loads and stresses in cylindrical stiffened shells while accounting for the nonlinear behavior and imperfection sensitivity exhibited by such shell structures. (A complete log of changes made in PANDA2 since 1987 is provided in the panda2.news file, distributed with the PANDA2 program.) PANDA2 performs stiffened-shellanalyses using a combination of exact and approximate analytical models of individual failure modes and some shell of revolution analysis models and has been demonstrated to provide accurate estimates for buckling loads and stresses. The next section provides a brief overview of PANDA2 analysis and optimization features with references to publications where details can be found.

This paper reports the results of a trade study that compared different stiffened-shell concepts designed with metallic and graphite-epoxy composite materials for a RLV liquid-hydrogentank design. A detailed description of the optimization problem, as formulated for the RLV tank structure design, is presented. Global optimization of stiffened shells with different combinations of materials and stiffener geometries were performed using PANDA2. Comparison of the optimum weights of the stiffened-shell concepts obtained numerically showed that the number of design variables optimized (design freedom permitted) and the conservativeness of the analysis models used to account for imperfections significantly influenced the optimum mass of design concepts. In addition, the effects of constraints imposed by the thermal protection system, fabrication techniques, and permeation of cryogenic liquid hydrogen on optimum design weight are illustrated.

The principal objective of the present paper is to shed light on the reasons for the mass differences between the different concepts. Specifically, we wish to see the degree to which these differences are caused by conservativeness introduced in the analysis to compensate for lack of experimental data on imperfection sensitivity of the concepts, the number of design variables optimized, and the nonstructural or manufacturing constraints imposed on the design.

# **Overview of PANDA2 Capabilities**

Designers of stiffened-shell structures have at their disposal a variety of analysis tools. These tools can be classified in increasing order of model fidelity and computational cost as collections of exact and approximate analytical models of individual stiffened-shell failure modes (e.g., PANDA<sup>5</sup> and PANDA2<sup>4,6–14</sup>), finite strip

analyses (e.g., VICONOPT<sup>15</sup>), and general purpose nonlinear shell finite element analyses (e.g., STAGS<sup>16,17</sup>). In the present work the large number of analyses needed for global optimization of multiple stiffener configurations and material concepts led us to choose PANDA2.

The PANDA2 computer program performs inexpensive structural analysis and optimization of stiffened and unstiffened-shell structures. Cylindrical and flat panels that have stiffeners with blade, hat, T, J, and Z-shaped cross sections in orthogonal (longitudinal and/or circumferential) directions or isogrid panels that have triangular stiffener arrangements can be analyzed. Sandwich shells with foam and honeycomb core can have additional longitudinal (stringers) and/or circumferential (ring) stiffeners. Isogrid-stiffened panels are permitted to have additional circumferential stiffeners. Several detailed papers describing PANDA2 analysis models, 4,7,10,13,14 theory, 5,6,10,11,12,14 optimization strategy, 10 and verifications 18–20 have been published. Here we present a summary of features of PANDA2 needed for understanding the present work.

PANDA2 employs relatively simple analysis tools, each designed to model and analyze a specific failure mode or mechanism of a stiffened panel. These include general instability of the entire panel, local buckling of skin regions between stringers and rings, panel buckling of the shell between ring stiffeners, buckling of stiffener webs and flanges and torsional buckling of stiffener (no web deformation) with skin participation, and torsional buckling of stiffeners (with web deformation) without skin participation and crippling of stiffener parts.

Structural analysis in PANDA2 uses closed-form analytical models (PANDA-type) models, as well as shell of revolution analysis of "a module" (Fig. 2) using BOSOR4<sup>21</sup> routines. A module includes the stiffener plus skin of width equal to the stiffener spacing. Figure 2 shows a single-panel module for a panel with T-shaped stiffener. The BOSOR4 analysis discretizes each segment of the stiffener module in the plane shown in Fig. 2 into number of elements and assumes harmonic variations in the orthogonal direction for its shell of revolution analysis. The BOSOR4 model (Fig. 2) uses both symmetry and antisymmetry boundary conditions at its end to estimate the local buckling load factor. Comparison with finite element analysis models have shown that this approximation is quite accurate. In the closed-formanalysis of local buckling, the stiffeners are with simple support boundary conditions. PANDA2 models for local buckling are valid as long as the shell structure has many repeating stiffeners in the axial and circumferential directions. Localized conditions at supports are not accurately reflected in PANDA2 analysis and require detailed finite element models. However, such details are not considered in the preliminary design trade studies. When the shell is subjected to normal pressure or if the axial load varies along the width of the panel, the entire width of the panel is analyzed using a discretized model with the stiffeners smeared.

PANDA2 can use linear or nonlinear theory to compute the static response of the panel subject to uniform normal pressure. The overall static response of the panel computed using a smeared representation is combined with the local response obtained from discretized single panel module to describe the stress state of the panel caused by applied pressure loads.

Initial imperfections that often result from manufacturing processes influence the buckling loads of cylindrical shells. PANDA2

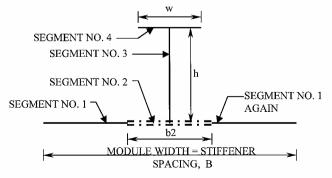


Fig. 2 Repeating element or "module" from a panel with T-shaped stiffeners used in BOSOR4 analysis.

conservatively assumes initial imperfection shapes to be the different critical mode shapes obtained from a bifurcation buckling analysis. The modal imperfections considered are general imperfection, local imperfection, and inter-ring imperfection. For complete cylindrical shells PANDA2 also considers an out-of-roundness imperfection that describes the ovalization in the cross section. Buckling loads with the effect of initial imperfections are computed using the lower value of two different theories: the closed-form analysis and Arbocz's Koiter-type theory.<sup>22–24</sup>

In practice, use of a fixed value for the imperfection amplitude can lead to nonconservative or overconservative designs. To overcome this, PANDA2 allows automatic adjustment of the buckling modal imperfection amplitude to a value that would be easily detectable by the most casual inspection. For a given value of imperfection amplitude, imperfection shapes with shorter (axial and circumferential) wavelengths are more easily detectable than imperfection shapes with long wavelengths because of their high curvatures. PANDA2 reduces the initial imperfection amplitude to a value that will give no more than 0.1 rad wall rotation.

The optimization is performed using the method of feasible directions as implemented in the ADS optimization subroutine developed by Vanderplaats.<sup>25</sup> Global optimization is based on automated random multiple restarts of line searches to locate the global optimum.

Papers published by Bushnell have presented validations of PANDA2 optimum designs with detailed nonlinear finite element analysis using the STAGS computer program<sup>18,19</sup> and with experimental test results.<sup>20</sup> The ring-stringer stiffened concept with T, J, and hat-shaped stiffeners is the most verified concept and hence is the least conservative in its approximations. Sandwich shell analysis features in PANDA2 were verified by Jiang<sup>26</sup> using more detailed models and accurate theory that include transverse shear deformations. He demonstrated that PANDA2 analysis was mostly accurate but occasionally conservative in its approximations. The approximations implemented in PANDA2 by its developer David Bushnell are based on expertise gained from his more than 30 years of experience in shell design. Bushnell has refined PANDA2 analysis models such that the errors in potentially optimal regions of the design space are smaller than elsewhere in the design space. In other regions the program typically uses more conservative approximations so that the buckling load limits are underpredicted for design purposes. The truss-core sandwich and isogrid-stiffenedshells have not been validated to the same extent as other concepts and therefore are implemented with more conservative analysis models.

Besides allowing global optimization by using simple models, PANDA2 also greatly facilitates the work of the designer. It does not require the construction of detailed finite element models of stiffened shells, and it automatically generates and analyzes the effect of various imperfection shapes.

# **Example Problem Description**

The principal objective of the present paper is to determine the degree extent to which the optimum weight estimates, obtained from structural through optimizations, are affected by conservativeness introduced in the analysis to compensate for lack of experimental data on imperfection sensitivity of the concepts and the number of design variables used in the optimization. To assess this issue, an example optimization problem has been selected that is typical of stiffened-shell design problems. The example problem involves the weight minimization of different cylindrical shells subject to load conditions that represent those experienced by a winged-body RLV liquid-hydrogen tank (Fig. 3). Propellant tanks that are an integral part of the RLVs have to carry structural loads. The minimum-weight structure is designed using buckling, strength (stress failure), and strain constraints.

The large cylindrical tanks (radius of 160 in.) have to be manufactured in smaller sections and assembled to form a whole tank. In this work we assumed that the tank would be manufactured as 300-in. barrel sections and assembled together with a ring frame at each joint. The cylindrical section with longitudinal (stringers) and/or circumferential (ring) stiffeners was optimized with simple-support boundary conditions at the ends along the longitudinal direction.



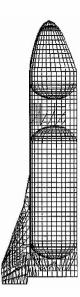


Fig. 3 Artist representation of a reusable launch vehicle concept and a half-symmetry finite element mesh of its primary structure used for obtaining design loads for panel optimization.

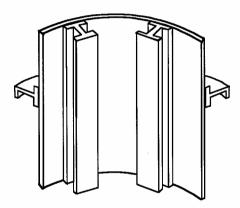


Fig. 4 Ring-stringer stiffened shell with internal T-shaped longitudinal stiffeners (stringers) and J-shaped circumferential stiffeners (rings).

# Stiffener Geometry and Material Selection

A schematic of typical stiffener cross-sectional shapes is shown in Fig. 1. In the present study several different stiffened-panel types were considered. The terminology used to describe the different stiffened shells and their classification based on stiffener shape and wall construction follows:

- 1) Ring-stringer stiffened shell is a biaxially stiffened cylindrical shell with T-shaped longitudinal (stringers) and J-shaped circumferential (ring) (Fig. 4).
- 2) Isogrid-stiffenedshell is a cylindrical shell that has rectangular blade stiffeners positioned along the circumferential and  $\pm 60$ -deg directions to the circumferential directions (Fig. 5).
- 3) Orthogrid stiffened shell is a cylindrical shell with orthogonal blade (or rectangular) stiffeners along the circumferential and longitudinal directions (Fig. 6).
- 4) Honeycomb-core sandwich shell is a cylindrical shell with honeycomb core sandwich construction for the wall stiffened in circumferential directions by J-shaped rings (Fig. 7).
- 5) Truss-core sandwich shell is a cylindrical shell with trapezoidal corrugated sandwich shells (Fig. 8).

Typical design variables in stiffened-panel design include the skin thickness, stiffener spacing (distance between stringers and/or rings), and widths and thicknesses of the different stiffener parts (segments). For composite materials the skin and segment thicknesses are replaced by the laminate stacking sequence and ply thicknesses. Additional details of the design variables optimized for the stiffened-shellconcepts just mentioned, such as the upper and lower bounds and optimum values, are presented in Ref. 27. The different stiffened shells investigated for the RLV design, their location

Table 2 Panel concepts, stiffener locations, and materials considered for the RLV liquid-hydrogen tank design

Panel type	Stringers	Rings	Material
Aluminum T-stringer and J-rings Aluminum isogrid-stiffened panel Aluminum orthogrid-stiffened panel Titanium symmetric sandwich: T-rings Titanium asymmetric sandwich: T-rings Titanium truss-core sandwich	Internal Internal blade isogrid Internal blade stiffeners None None Hat-shaped corrugation	External External J-rings None Internal Internal N/A	Al-2219 T87 Al-2219 T87 Al-2219 T87 Ti-6Al-4V Ti-6Al-4V Ti-6Al-4V

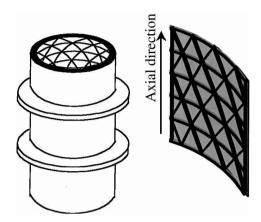


Fig. 5 Isogrid-stiffened cylindrical shell with internal isogrid and external rings with isogrid pattern oriented along circumferential direction for increased bending stiffness in hoop direction.



Fig. 6 Orthogrid-stiffened cylindrical shell with identical blade stiffeners along longitudinal and circumferential directions at different spacing.

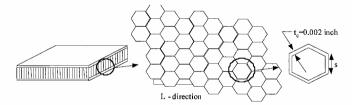


Fig. 7 Honeycomb-core sandwich laminate with honeycomb-core L-direction oriented along axial direction of the shell. Honeycomb-core cell size s and ribbon thickness  $t_c$  are design variables.

(internal vs external), and materials used in the design of RLV liquid hydrogen (cylindrical) tanks are summarized in Table 2. The selection of stiffener types and their positioning (external vs internal) was based on manufacturing considerations, type of thermal protection system (TPS), and the TPS attachment method used for the stiffened cylindrical tank.

In addition to considering several different stiffened-panel concepts, a variety of materials, which cover a fairly wide range of launch vehicle material options, were considered in the present study. Both metallic and composite materials were considered, and the properties of these materials are listed in the Appendix.

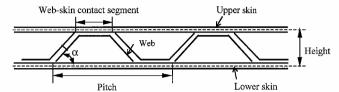


Fig. 8 Cross section of a truss-core sandwich panel with continuous trapezoidal-shaped corrugation.

#### Requirements Imposed for Metallic Designs

The aluminum and titanium alloys used for metallic concepts have approximately the same specific stiffness, but the titanium alloy has 45% higher specific strength. The ring-stringer, isogrid-, and orthogrid-stiffenedshells were designed using aluminum alloy (Al 2219-T87), and the honeycomb core and truss-core sandwich were designed using titanium alloy (Ti-6Al-4V). The titianium alloy is not used for grid-stiffened or ring-stringerstiffened shells because of the high machining costs associated with the material. However, titanium lends itself to being produced in sheets and hence makes it easy to use for in sandwich shells.

The ring-stringerand isogrid-stiffenedshell were optimized with closely spaced external ring stiffeners that will be used for attaching the TPS tiles. The isogrid-stiffening pattern was oriented such that one of the isogrid stiffeners coincided with the circumferential direction of the shell to provide higher stiffness in the circumferential direction. Orthogrid-stiffened shells could not be optimized with additional external rings because PANDA2 does not have a separate model for orthogrid-stiffened shells. The orthogrid-stiffened shells were simply modeled as special case of ring-stringer stiffened shells, where longitudinal and circumferential stiffener dimensions were kept identical by design variable linking. However, stiffener spacing variables were not linked.

Internal rings were used for the sandwich case because the TPS chosen for this concept required a smooth outer surface. Optimizations were performed with fixed and variable ring spacing to ensure that the final design was not a local optimum. The facesheets and their expanded ribbon core in honeycomb core sandwich shells are made of a titanium (Ti-6Al-4V) alloy. Manufacturing requires that core cell wall thickness  $t_c$  (commonly referred to as ribbon thickness) be at least 0.002 in. PANDA2 optimizations can also vary the core diameter and cell wall thickness. Sandwich walls with symmetric and asymmetric constructions were considered. The unfilled honeycomb core is typically evacuated in RLV applications to minimize the thermal conductivity through the wall. The thermal resistance provided by the core of the sandwich concept can provide further weight savings because of reduced weight of the required thermal insulation on the cryogenic tank when the core is sufficiently thick. The truss-core stiffened-shell concept was optimized without ring stiffeners because of PANDA2's modeling limitation.

#### **Requirements Imposed for Composite Designs**

PANDA2 optimizations using composite materials were performed for ring-stringer, honeycomb-core sandwich, and truss-core sandwich stiffened-panelconcepts (Table 3). Isogrid- and orthogrid-stiffened shells were not designed for composite materials because of manufacturing limitations. Filament winding techniques developed for manufacturing isogrid-stiffened shells could not be used for the RLV tank because it is not possible to autoclave such a large size tank (320 in. diam) as one piece. Ring spacing was included as

Table 3 Composite stiffened-shell concepts optimized for the RLV liquid-hydrogen tank design

Panel type	Stringers	Rings	Material
Composite T stringer and T-rings	External	External	IM7/977-2
Composite symmetric sandwich: T-rings	None	Internal	IM7/977-2
Composite asymmetric sandwich: T-rings	None	Internal	IM7/977-2
Composite truss-core sandwich	External corrugated skin	None	IM7/977-2

Table 4 Laminate layups used for the different composite shells<sup>a</sup>

Concept	Shell segment	Layup design	Laminate thickness, in.		
	Including thermal considerations				
Stringer and ring stiffened	Wall	$[(+65/-65)_3]_S$	0.06		
	stiffeners	$[+45/-45/0_{n1}/-45/+45/0_{m1}]_{S}$	0.055-0.100		
Honeycomb-core sandwich	Inner facesheet	$[(+65/-65)_3]_S$	0.06		
panel with external rings	Outer facesheet	$[(+65/-65)_{n1}]_{S}$	0.02-0.06		
	Ring stiffeners	$[+45/-45/0_{n2}/-45/+45/0_{m1}]_{S}$	0.055-0.100		
Truss-core panel	Skin	$[(+65/-65)_3]_S$	0.06		
	corrugation	$[+45/-45/0_{n1}/-45/+45/0_{m2}]_{S}$	0.055-0.100		
	Ignoring	thermal considerations			
Stringer and ring stiffened	Wall	$[+45_{n1}/90_{n2}/-45_{n1}/0_{m1}]_{S}$	0.035-0.110		
	stiffeners	$[+45_{n3}/0_{n4}/-45_{n5}/90_{m2}]_{S}$	0.035-0.110		
Honeycomb-core sandwich	Inner facesheet	$[+45_{n1}/90_{n2}/-45_{n1}/0_{m1}]_{S}$	0.035-0.110		
panel with external rings	Outer faceseet	$[+45_{n3}/90_{n4}/-45_{n3}/0_{m2}]_{S}$	0.035-0.110		
	Ring stiffeners	$[+45/90_{n5}/-45/0_{m3}]_{S}$	0.035-0.110		

<sup>&</sup>lt;sup>a</sup>Subscripts  $n_i$  and  $m_i$  in the laminate stacking sequence indicate the plies whose thickness are optimized. To ensure that no more than four contiguous plies have the same orientation,  $n_i$  has to be less than four, whereas  $m_i$  is limited to two because of its location at the symmetry plane.

a design variable for ring-stringer stiffened and honeycomb-core sandwich panel optimizations. Composite truss-core shells were also optimized for a 300-in. barrel length without ring stiffeners. Sandwich shells were optimized with symmetric and asymmetric facesheet constructions. The composite material used was the IM7/977-2 graphite epoxy system (Table A2).

Optimizing ply orientations in preliminary design is not recommended for a number of reasons. Composite laminates have failure modes that are difficult or expensive to model and analyze during preliminary design optimization. Furthermore, in a preliminary design optimizations are generally performed using a small number of load cases. Optimizing ply orientations using a small set of design load cases and simple analyses can produce laminates that fail in neglected failure modes or loading conditions.

Optimizations of composite shells were performed for fixed ply angles and laminate stacking sequences (provided by NASA Langley Research Center) for which strain allowables were obtained from tests performed at room and cryogenic temperatures. The stress limits of the composite materials used in the optimization are shown in Table A2. All laminates used for the stiffened-panel design were balanced and symmetric. In addition to sizing optimization for fixed laminate thickness and stacking sequence, for some cases individual ply thicknesses were optimized keeping the same stacking sequence. For these cases optimization required two iterations. In the first iteration the ply thickness and size variables were optimized as continuous variables. The optimum ply thickness from the continuous optimization was rounded to the nearest integer multiple of pre-preg ply thickness. The panel design variables, excluding ply thicknesses, were once again optimized to ensure that the ply thickness rounding did not result in suboptimal or infeasible designs. Table 4 shows the laminates used in the optimization and the ply thicknesses that were used as variables.

Laminates that have plies oriented along both longitudinal and circumferential directions can develop matrix cracking caused by the residual thermal stresses. Therefore, an angle-ply laminate with  $\pm 65$ -deg plies was chosen for the wall and facesheet laminates. This ply orientation reduced the thermal stresses developed in the laminate caused by thermal mismatch and provided some stress margin for the mechanical loads. Stacking sequence of laminates used in liquid-hydrogentanks obtained using deterministic and probabilistic optimization are compared by Qu et al.  $^{28}$ 

Composite panel designs have an additional requirement when used for liquid-hydrogentanks. To limit liquid-hydrogenpermeation to acceptable levels, the wall laminate thickness of stiffened shells

or thickness of facesheet in the case of sandwich was required to be at least 12 plies (0.06 in. thick). This is a severe constraint for sandwich constructions because they do not need such thick facesheets for structural requirements. Asymmetric sandwich laminates with different facesheets were therefore optimized to investigate their weight saving. For asymmetric designs only the inner facesheet has to satisfy the 12-ply requirement; the outer facesheet can be thinner. In such designs each facesheet is balanced and symmetric. Shells were optimized with the same initial imperfection values as those used for metallic shells.

We also optimized ring-stringer, honeycomb sandwich, and truss-core sandwich stiffened shells with laminates that were not restricted by the thermal microcracking constraint. The laminate was optimized using plies of 0-,  $\pm 45$ -, and 90-deg orientations. The 0-deg direction corresponds to the axial direction in the stacking sequence of laminates for wall and the axial stiffener and corresponds to the circumferential direction in the stacking sequence of the ring stiffener laminate.

# Design Loads, Safety Margins, and Initial Imperfections

Two load cases were used: 1) internal proof pressure of 35 psi, which is critical for strength, and 2) axial compressive load  $N_x = 1000$  lb/in., with an internal (stabilizing) pressure of 5 psi, which is critical for buckling.

Safety margins used in the optimization were 1.4 for general buckling, 1.2 for local and stiffener buckling, and 1.2 for stress failure. The safety factor for local buckling is lower because buckling of the skin or wall often does not greatly affect the structural integrity of the launch vehicle structure. Using different safety factors for the different modes also minimizes mode interactions in the optimized designs. Stress allowables for the different materials are shown in the Appendix. Postbuckling strength constraints are not addressed here. The effect of including local postbuckling of skin on the optimum designs is presented in Refs. 29 and 30.

The following initial imperfection amplitudes were chosen for this study: 1) general imperfection—0.8 in. (0.5% of cylinder radius), 2) out-of-roundnessimperfection—0.8 in. (0.5% of the cylinder radius), 3) interring imperfection—set equal to 100% of skin thickness, and 4) local imperfection—set to 10% of the cylinder wall thickness.

The preceding selected values for initial imperfections are typical of those used by the pressure vessel design community. PANDA2 reduces the specified values if it detects that the wall rotation exceeds 0.1 rad. For sandwich shells facesheet wrinkling is affected

by the initial waviness of the facesheet. The ratio of initial facesheet waviness to facesheet wrinkling half-wavelengthwas assumed conservatively as  $10^{-3}$  based on information available from other design studies  $^{13}$  of metallic sandwich construction. The same was assumed for composite design because of a lack of data on composite sandwich constructions.

Once again, the principal objective of the present paper is to determine the degree to which the weight estimates, obtained from preliminary design optimization programs, are affected by the conservativeness introduced in the approximate analysis models to compensate for lack of experimental data on imperfection sensitivity of the concepts.

## **Optimization of Metallic Stiffened Shells**

The weights of stiffened shells optimized with and without geometric imperfections are shown in Table 5. The stiffener weight fractions for both cases are calculated and compared. The active constraints or failure modes of the optimum metallic designs are listed in Table 6. The optimum values of the variables of the designs corresponding to the weights in Table 5 are presented in Refs. 27 and 29

In the absence of geometric imperfections, the honeycomb-core sandwich concept was the most efficient, and the grid-stiffened concepts were the heaviest. A significant portion of the optimum weight is caused by the strength required to carry the 35-psi internal pressure load. The panel weights of cylinders sized to carry this 35-psi pressure load alone are 1.324 and 0.910 lb/ft², respectively, for aluminum and titanium alloys used in this study. Honeycomb-core sandwich shells with symmetric facesheets did not have any active buckling constraints (Table 6) and weighed 1.110 lb/ft², which is higher than the weight computed for an unstiffened shell. This is because of the small extra weight contributed by the minimum thickness core and ring stiffeners. The asymmetric sandwich construction marginally reduced the optimum weight, but also reduced the safety margin on

global buckling load factor. The low optimum weight of sandwich shells can be attributed to the 45% higher specific strength of the titanium alloy material used for sandwich compared to aluminum used for stiffened-shellconcepts. The weight of the symmetric sandwich shell concept increases to 1.588 lb/ft² (close to the weight of ring-stringer stiffened shell) when its facesheets are replaced by aluminum alloy. Sandwich constructions also do not have stiffener buckling modes that affect stiffened shells. Because the honeycomb core also has thermal insulation properties, the weight of the core is reported separately for sandwich designs. All or some fraction of this weight can be considered as the thermal insulation weight for RLV applications. If the weight of the sandwich core is not included as structural weight, the sandwich concepts are extremely light compared to other concepts.

The truss-core sandwich construction was significantly heavier than the honeycomb-core sandwich construction. This might be caused by the PANDA2 limitation of not allowing ring stiffeners for truss-core sandwich shells. Therefore, the cylindrical truss-core shell is designed without ring stiffeners. Additional details of truss-core panel designs optimized for different lengths with clamped boundary conditions and the PANDA2 treatment for bending boundary-layer length at the clamped ends are presented in Ref. 27. The design obtained here without rings in the barrel section is a conservative estimate.

The optimum values of the corrugation angle in truss-core sandwich shells are 57.0 and 62.3 deg when optimized for perfect and imperfect shells, respectively. Most conventional truss-core shells are designed for a 45-deg corrugation angle, which maximizes the transverse shear stiffness in the circumferential direction. It appears that the optimum design sacrifices some transverse shear stiffness to use the corrugation in carrying the hoop stress, which results in substantial weight reduction.

Geometric imperfections introduce load eccentricity that increases prebuckling bending and thereby causes local and stiffener

	Without the effect of	fimperfections	With the effect of imperfections			
Stiffened-panel type	Weight of panel, lb/ft², and core weight for sandwich designs, lb/ft²	Stiffener weight fraction, %a	Weight of panel, lb/ft², and core weight for sandwich designs, lb/ft²	Stiffener weight fraction, %	Weight incrfease caused by imperfections, %	
Ring-stringer stiffened	1.600	13.3	1.840	24.0	15.0	
Isogrid	1.537	8.2	2.409	36.2	56.7	
Orthogrid	1.538	10.0	2.147	38.2	39.6	
Honeycomb-core sandwich	1.110	8.67	1.323	23.4	19.2	
with symmetric facesheets	(0.082)		(0.289)			
Honeycomb-core sandwich with asymmetric facesheets	1.095 (0.082)	9.5	1.320 (0.291)	24.9	20.5	
Truss-core sandwich	1.468	29.8	1.705	39.4	16.1	

Table 5 Weight comparisons of metallic stiffened-panel concepts optimized with and without the effect of geometric imperfections

Table 6 Active constraints for metallic stiffened-panel concepts

Stiffened-panel type	Shells designed without the effect of imperfections	Shells designed with the effect of imperfections
Ring-stringer stiffened	Stress (pressure), local buckling, wide column buckling	Stress (pressure), stress, local buckling, stringer web and top flange buckling
Isogrid	Stress (hoop pressure), local buckling, stiffener web buckling	Local buckling, stiffener web buckling
Orthogrid	Stress (hoop pressure), local buckling, stringer web buckling	Stress (pressure), stress in stringer and ring webs, local, general and stringer web buckling
Honeycomb-core sandwich with symmetric facesheets and optimized core	Stress (pressure)	Stress (pressure), general buckling, local buckling, ring web buckling, ring top flange buckling
Honeycomb-core sandwich with asymmetric facesheets with optimized core	Stress (pressure), general buckling	Stress (pressure), general buckling, local buckling, ring web and top flange buckling
Truss-core sandwich	Stress (pressure), corrugation web buckling	Stress (pressure), global buckling, upper skin buckling, corrugation web buckling

<sup>&</sup>lt;sup>a</sup>Includes honeycomb- and truss-core weights for sandwich concepts.

buckling. This results in a substantial weight increase for the isogridand orthogrid-stiffened-shell concepts and smaller increases for the other concepts. The large increase in weight for grid-stiffenedshells is caused by the use of blade stiffeners that are more susceptible to buckling and imperfection sensitivity. The high imperfection sensitivity of the isogrid stiffeners might be caused by PANDA2 modeling them as smeared, leading to isotropic equivalent shell for general buckling calculations and because of the effect of prebuckling bending. PANDA2 analysis of grid-stiffenedshells includes the prebuckling bending of initially imperfect shells that causes stress redistribution in various small parts of the stiffeners, thereby giving rise to local buckling of stiffener webs. The increased imperfection sensitivity is in contrast to work by Vasiliev et al.,<sup>31</sup> which showed that anisotropic lattice structures are highly imperfection insensitive. It is possible that the PANDA2 treatment is conservative and reflects the scarcity of data on imperfection sensitivity of grid-stiffened shells.

For all four stiffening concepts optimization produced flimsy rings that were less than 10% of total weight. This is because rings do not offer much help in reducing buckling loads under pure axial compression. Other load cases, such as overall bending of the cylinder or nonsymmetric external pressure from flight loads, have to be used to size the ring stiffeners. However, including ring stiffeners in the optimization is useful because it can help decrease the imperfection sensitivity of the shells by preventing simultaneous occurrence of different buckling modes at identical load level (as in unstiffened cylinders).

# **Optimization of Composite Stiffened Shells**

Table 7 shows the optimum weights, stiffener weight fractions of optimum composite stiffened shells, and the effect of considering initial imperfections. The active constraints of the optimum designs are listed in Table 8. Details of values of the design variables and laminate stacking sequence for optimum designs are presented in Refs. 27 and 29.

Surprisingly, for composite materials the ring-stringer stiffened panel was much lighter than the honeycomb-core sandwich panel. A significant portion of the weight of ring-stringerstiffened concept is in the stiffener weight. In particular, in designs optimized with thermal consideration the low axial stiffness of the wall laminate with  $\pm 65$ -deg plies is compensated by the presence of closely spaced and heavier axial stiffeners. For metallic shells the ring-stringer concept was 40% heavier than the symmetric sandwich concept, whereas for the composite shells the sandwich was almost 50% heavier. For truss-coreshells the PANDA2 limitation of not allowing ring stiffeners contributes to a weight increase.

In the case of symmetric sandwich panel, the lower bound imposed (0.06 in. or 12 plies) on the facesheet thickness to avoid hydrogen permeation was higher than that required for structural integrity. The asymmetric sandwich panel has a lower weight, as the outer facesheets had only eight plies (0.04-in. thickness). The asymmetric sandwich panel optimizations were performed for inner facesheet laminates of  $[(\pm 65)_2]_s$  layups that had eight plies, which led to a 10% weight reduction. To compensate for the bending stiffness reduction in asymmetric sandwich construction, the optimization increased the core thickness. The core thickness of the asymmetric sandwich panel design optimized using the effect of initial imperfections (1.206 in.) was 12% thicker when compared to the optimum symmetric sandwich design (1.076 in.). However, asymmetry can have other disadvantages, such as manufacturing difficulties and introduction of curvature caused by thermal strains that can act as geometric imperfections in shells. Without additional consideration of such issues, asymmetric sandwich panel constructions cannot be recommended.

In comparing the minimum weight designs of sandwich concepts with other designs, one also needs to consider the potential weight savings that can result from the insulation provided by the sandwich core. If the weight reduction of the thermal insulation is comparable to the sandwich core weight, then sandwich shells can become

Table 7 Optimum weight and stiffener weight fractions of composite stiffened-panel concepts optimized without and with initial imperfections

	Shells designed we effect of imper-		Shells designed effect of imper		
Stiffened-panel type	Weight of panel, lb/ft <sup>2</sup> , and core weight for sandwich designs, lb/ft <sup>2</sup>	Stiffener weight fraction, %a	Weight of panel, lb/ft <sup>2</sup> , and core weight for sandwich designs, lb/ft <sup>2</sup>	Stiffener weight fraction, %a	Weight increase caused by geometric imperfections, %
	Including there	nal considerations;	laminate with ±65-deg plie	es	
Ring-stringer stiffened	0.822	40.0	0.911	45.9	10.8
Honeycomb-core symmetric	1.168	15.7	1.362	39.7	16.6
sandwich	(0.164)		(0.353)		
Honeycomb-core asymmetric	1.003	18.2	1.237	33.7	23.3
sandwich	(0.168)		(0.395)		
Truss-core sandwich	1.279	23.0	1.299	24.2	1.6
	Ignoring thermal con	siderations; lamina	tes with 0-, $\pm$ 45-, and 90-d	leg plies	
Ring-stringer stiffened	0.802	33.5	1.206	55.8	50.38
Honeycomb-core symmetric	1.250 <sup>b</sup>	14.6	1.358	21.4	8.6
sandwich	(0.164)		(0.238)		
Honeycomb-core asymmetric	0.975	7.4	1.169	22.76	19.9
sandwich	(0.164)		(0.282)		

<sup>&</sup>lt;sup>a</sup>Includes honeycomb- and truss-core weights for sandwich concepts.

Table 8 Active constraints of composite stiffened-panel concepts optimized without and with initial imperfections, including thermal considerations

<b>F</b>				
Stiffened-panel type	Shells designed without initial imperfections	Shells designed with initial imperfections		
Ring-stringer stiffened	Fiber tensile stress (pressure)	Fiber tensile stress (pressure), stringer buckling, rolling of stringers		
Honeycomb-core symmetric sandwich	Stress in transverse to fibers (pressure)	Local buckling, ring rolling		
Honeycomb-core asymmetric sandwich	Stress in transverse to fibers (pressure), global buckling	Fiber tensile stress (pressure), local buckling, global buckling, ring rolling		
Truss-core sandwich	Fiber tensile stress (pressure), upper facesheet buckling	Fiber tensile stress (pressure) upper facesheet buckling, global buckling		

<sup>&</sup>lt;sup>b</sup>The small increase in weight compared to design with thermal considerations is caused by the requirement of at least one ply with 0-,  $\pm$ 45-, and 90-deg orientation in the laminate.

as competitive as other concepts for weight considerations. In fact, when the core weight is not included as part of the structural weight the asymmetric sandwich shells designed with initial imperfections are comparable in weight to ring-stringer stiffened-panel design (0.911 lb/ft²), as shown in Table 7. The asymmetric panel designed with imperfections is lighter (0.842 lb/ft²) than the ring-stringer stiffened concept for the designs with wall laminates having 65-deg plies. Truss-core panel has optimum weight comparable to sandwich shells with symmetric construction and optimized core thickness.

# Comparison of Optimum Solutions for Metallic and Composite Stiffened Shells

This section presents a comparison of the optimum weight estimates for both metallic and composite stiffened shells, and an assessment of the degree to which the weight estimates of optimized designs are affected by factors such as analysis conservativeness, number of optimization variables permitted, and the nonstructural/manufacturing constraints are discussed

The relative ranking of stiffened-panel concepts based on optimum weight from preliminary optimization was quite different for the metallic and composite material designs. Although the honeycomb-core sandwich panel was the most efficient concept for metallic designs, the ring-stringer stiffened concept was the most efficient concept for composite design. The weight reduction for the ring-stringer stiffened shell was approximately 50% when the material is changed to composite material. The weight reduction is smaller for sandwich shells because the metallic designs were obtained using a titanium alloy of higher specific strength. Composite sandwich shells are further disadvantaged by the choice of the facesheet laminate (with plies in  $\pm 65$ -deg direction) that has low stiffness in the axial direction. The thermal stress considerations for reducing microcracking required the ply-angle variations to be limited to less than  $\pm 25$  deg.

The optimum weights of ring-stringer stiffened and honeycomb-core sandwich shells with laminates that did not consider thermal microcracking are presented in Table 7. The thickness of the plies in the 0-, 45-, and 90-deg orientations were included as optimization variables. For sandwich shells, when thermal considerations were ignored, lower weight optimum designs were obtained as a result of the increased in-plane stiffness of facesheets in axial direction (Table 7).

The angle-ply laminates that satisfy thermal considerations were more efficient in the case of ring-stringer stiffened shells. The bending stiffness of the wall laminate for resisting buckling along the axial direction was increased by using axial stiffeners. The angleply laminate wall is efficient for carrying the circumferential stresses as a result of the internal pressure loads, whereas the stiffeners are efficient for increasing bending stiffness required for preventing buckling caused by axial loads. This results in efficient utilization of the material. Because the stiffeners provide most of the required bending stiffness, the ring-stringer stiffened shell is less sensitive to imperfections in the shell wall. The ring-stringer stiffened shell optimized using laminates obtained without thermal considerations has almost identical weight as the design with  $\pm 65$ -deg ply angle laminate when there are no initial imperfections (Table 7). However, when initial imperfections were introduced the weight increased by 50%. The increased wall participation in resisting buckling loads makes the design more imperfection sensitive. Laminate stacking sequences of the optimum composite designs with and without thermal considerations are presented in Table 9.

The effect of design freedom allowed in optimization on the optimum weight and imperfection sensitivity is illustrated for the ring-stringer stiffened shell in Table 10. The optimum weight of designs and sensitivity to initial imperfections decreased as the design freedom is increased. The weight reduction is similar to that observed for sandwich shells when design freedom is increased (symmetric vs asymmetric laminates). However, for sandwich construction increasing design freedom increased the imperfection sensitivity. This imperfection sensitivity in the case of ring-stringer stiffened shell stems from the presence of the two different load paths for the two load cases used in the design. The wall laminate was optimized for the pressure load case and the stringers for the axial compression load case. This separation in load-carrying capacity was not possible for the sandwich shell.

The preliminary design did not consider details such as thermal stress-induced failure in wall laminate as a result of mismatches in thermal expansion coefficients between stiffeners and wall laminate, local stresses at stringer edges induced by Poisson's ratio mismatch, interlaminar shear stresses, or matrix microcracking under cryogenic loads. Detailed analysis of the final optimum designs must be performed to assess the viability of the preliminary designs.

Table 9 Optimized stacking sequence of composite facesheets in honeycomb-core sandwich shells optimized with different facesheets laminates

Design type	Stacking sequences of laminates chosen with thermal considerations	Stacking sequences of laminates chosen ignoring thermal considerations
Symmetric sandwich with optimized core	IFS <sup>a</sup> : $[\pm 65]_{3S}$ OFS <sup>b</sup> : $[\pm 65]_{3S}$ RNG <sup>c</sup> : $[\pm 45/0_2/\mp 45/0_{1/2}]_S$	IFS: $[45/90_3/-45/0_{3/2}]_s$ OFS $[45/90_3/-45/0_{3/2}]_s$ RNG: $[\pm 45/0/\mp 45/0_{1/2}]_s$
Asymmetric sandwich with optimized core	IFS: $[\pm 65]_{3S}$ OFS: $[\pm 65]_{2S}$ RNG: $[\pm 45/0/\mp 45/0]_{S}$	IFS: $[45_2/90_2/-45_2/0]_s$ OFS $[45/90/-45/0_{1/2}]_s$ RNG: $[\pm 45/0/\mp 45/0_{1/2}]_s$

<sup>&</sup>lt;sup>a</sup>IFS = inner facesheet. <sup>b</sup>OFS = outer facesheet. <sup>c</sup>RNG = ring stiffener laminates.

Table 10 Optimized stacking sequence of laminates designed for composite ring-stringer stiffened shells optimized with different levels of design freedom

Design variables optimized	Without initial imperfections			With initial imperfections		
	Panel weight	Stiffener weight fraction, %	Wall and stiffener laminate	Panel weight	Stiffener weight fraction, %	Wall and stiffener laminate
Stiffener spacing and sizing	0.839	41.3	$[(+65/-65)_3]_S$ $[\pm 45/0/\mp 45/0]_S$	1.019	51.7	$[(+65/-65)_3]_S$ $[\pm 45/0/\mp 45/0]_S$
Stiffener spacing and sizing and thickness of plies in stiffener laminate	0.826	40.4	$[(+65/-65)_3]_S$ $[\pm 45/0_2/\mp 45/0_1]_S$	0.911	45.9	$[(+65/-65)_3]_S$ $[\pm 45/0_4/\mp 45/0_2]_S$
Stiffener spacing and sizing and thickness of plies in wall laminate	0.826	35.4	$[+45/90_4/-45/0]_S$ $[+45/0/-45/90]_S$	1.333	60.0	$[+45_2/90_2/-45_2/0_2]_S$ $[+45/0/-45/90]_S$
Stiffener spacing and sizing and thickness of plies in wall and stiffener laminates	0.802	33.5	$[+45/90_4/-45/0]_S$ $[+45/0/-45/90]_S$	1.206	50.38	[+45 <sub>2</sub> /90 <sub>3</sub> /-45 <sub>2</sub> /0] <sub>S</sub> [+45/0 <sub>4</sub> /-45/90] <sub>S</sub>

#### **Conclusions**

Optimizations of metallic and laminated composite stiffened shells were performed for a reusable-launch-vehicle liquidhydrogen tank using PANDA2 under strength critical and stability critical load cases. The weight estimates of stiffened shells obtained from optimization that included initial imperfections were compared to those obtained without including initial imperfections. The optimization results were useful for identifying different factors that influence the optimum weight of stiffened-shell designs. For metallic shells differences in weights observed between the stiffened and sandwich shells were caused by the differences in the specific strength of the materials used. Because of the high specific strength of the titanium material used, the honeycomb-core sandwich design was the lightest among metallic designs. A significant fraction of the stiffened-shell weight was in weight of the skin designed to withstand a 35-psi internal pressure. Buckling constraints were satisfied with a small increase in the weight. However, when initial imperfections were considered the designs resulted in significantly different optimum weights. The grid-stiffened and truss-core sandwich shells exhibited a large increase in their optimum weight. The heavier weights of grid-stiffened shells were partly caused by the use of blade stiffeners that have small torsional stiffness. However, the significant factor was caused by conservativeness in the PANDA2 imperfection sensitivity analysis of grid-stiffened shells. The grid-stiffened and truss-core sandwich shells are concepts in PANDA2 analysis that have been the least validated and fine tuned with experimental results. Because of the lack of information on imperfection sensitivity information for grid-stiffened and truss-core concepts, the approximations used in the preliminary analysis are made more conservative (underpredict buckling strength) to ensure safe, although heavier, designs.

The composite designs were significantly affected by thermal considerations, which dictated designs with laminates having only  $\pm 65$ -deg plies. However, this was found to be of significant advantage in the design of the ring-stringer stiffened shell. The low axial stiffness of skin laminates with  $\pm 65$ -deg plies required larger and more closely spaced axial stiffeners to resist buckling caused by axial load. Because the axial stiffeners carry a significant portion of the axial load, the ring-stringer stiffened shell was less imperfection sensitive. In contrast, ring-stringer stiffened shells designed without thermal considerations (laminates having 0-,  $\pm 45$ -, and 90-deg plies) show large weight penalties caused by initial imperfections.

The sandwich shells suffered from the requirement of a minimum of 12 plies to reduce hydrogen permeation through the tank wall. Without this requirement it was possible to obtain sandwich shells that were of comparable weight to the stiffened shells. Relaxing this constraint for one of the facesheets resulted in lighter asymmetric sandwich designs. However, asymmetric designs are less robust when subject to small variations from design conditions and introduce manufacturing difficulties.

Composite stiffened shells permit optimization on more design variables because the individual ply thickness can be changed in addition to the stiffener sizes and spacing. It is shown that increasing the number of optimization variables (design freedom) can reduce the optimum weights. However, increased design freedom also increases the imperfection sensitivity of the optimum designs and makes them less robust.

As observed for the metallic shells, the truss-core sandwich construction was substantially heavier than the stiffened or sandwich-shell composite designs. This is caused by the model deficiencies and the conservatives of PANDA2 analysis for this concept. More experimental imperfection sensitivity data must be generated for grid-stiffened and truss-core sandwich shells to reduce the conservativeness of approximate analysis models for such designs.

The results presented here lead us to conclude that preliminary design codes which use efficient approximation are very useful for design space exploration, for identification of potential designs, and for identification of factors that influence the performance (e.g., weight) of each concept. However, the optimization results from preliminary design codes must not be used for the final selection of the design concept or ranking of the concepts based on performance,

without carefully considering the quality of the analysis approximations for the different designs.

## **Appendix: Material Properties**

#### Metallic Alloys

Two metallic lightweight alloys were used. The selection was based on structural requirements and the performance of the material at cryogenic temperatures. The material properties are provided in Table A1.

# **Laminated Composites**

Composite stiffened panel constructions were designed using the IM7/977-2 graphite-epoxy system. The material properties are provided in Table A2.

#### Honeycomb-Core Sandwich

The properties of the honeycomb core, namely, transverse modulus  $(E_c)$  and shear moduli  $(G_{xz}, G_{yz})$ , are calculated in PANDA2<sup>13</sup> using simple geometric relations. The following empirical relations obtained from the manufacturer (Hexcel Corp.) were used in this study to calculate the core crushing and core shear strengths:

$$F_{\rm cc} = 2.31 (\tilde{n}_c'/\tilde{n}_c)^{1.464} (F_{\rm tu}/c^{0.16})$$
 (A1)

$$F_{\rm csL} = 1.307 (\tilde{n}'_c/\tilde{n}_c)^{1.34} (F_{\rm tu}/c^{0.44})$$
 (A2)

$$F_{\rm csW} = 0.8F_{\rm csL} \tag{A3}$$

For hexagonal expanded honeycomb the core density can be obtained using

$$\rho_c' = \frac{8}{3} (t_c/d_c)$$

The expressions for crushing and shear strengths just shown are valid only for a sandwich core thickness of 1 in. Core crushing and shear strengths decrease with increases in thickness: doubling the thickness decreases the crushing and shear strengths by 10 and 35%, respectively. The manufacturer provided a table of strengths as a function of thickness, and this was used as input in PANDA2. The PANDA2 analysis interpolates the values in the table for intermediate values of the core thickness.

Table A1 Material properties of lightweight alloys used for metallic RLV tanks at 50° F

Property	Al 2219-T87	Ti-6Al-4V
Density, lb/in. <sup>3</sup>	0.1	0.16
Elastic modulus, Msi	10.7	16.5
Shear modulus, Msi	4	6.3
Poisson's ratio	0.34	0.32
Yield stress, psi	58,000	135,000

Table A2 Material properties for IM7/977-2 fiber reinforced polymer composite material (at 190°F)

Property	Value
Density, lbs/in. <sup>3</sup>	0.057
Elastic modulus in the fiber direction, Msi	21.5
Elastic modulus in the transverse fiber direction, psi	1.08
In-plane shear modulus $G_{12}$ , psi	0.6
Out-of-plane shear moduli $G_{13}$ , $G_{23}$ , Msi	0.51
Small Poisson's ratio	0.1507
Allowable tensile (compressive) stress in the	129
fiber direction, Ksi	(176.3)
Allowable tensile (compressive) stress in the	21.6
transverse fiber direction, Ksi	(36.0)
Allowable shear stress, Ksi	10.9
Thermal expansion coefficient along fibers, $\times 10^{-6}$ /°F	-0.11
Thermal expansion coefficient transverse to fibers, $\times 10^{-6}$ f° F	17.0
Ply thickness, in.	0.005

# Acknowledgments

The support of the Thermal Structures Branch at NASA Langley Research Center through Grant NAG-1-1808 is gratefully acknowledged. The financial support of Politecnicodi Bari for Luciano Lamberti's visit to the University of Florida is acknowledged. The authors thank David Bushnell for his help with using and understanding PANDA2.

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M. S. Lake Associate Editor